Frequency Domain Quasi Maximum Likelihood Identification of Low Order Aeroservoelastic Models from Flight-Test Data

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Background and Motivation

Low Order Equivalent System (LOES)

- From handling qualities analysis
- Traditionally simplifying complex control law and plant
- More easily understood form
- Extend LOES to a complex model due to aeroelasticity

Maximum likelihood (Filter Error) System Identification

$$\boldsymbol{Z} = \boldsymbol{H}_{loes}(\boldsymbol{U} + \boldsymbol{W}) + \boldsymbol{V}$$

- There are a lot of parameters
  - Estimate noise parameters
  - Extra outputs mean extra parameters to estimate
- Usually assume noise model to simplify
  - Output Error and Equation Error
  - Results in biased estimates of the parameters
- Sensing flexible aircraft have many outputs
  - Quasi maximum likelihood exploits redundancy of outputs

Definition of Model

Low order system

$$\boldsymbol{H}_{loes}(s|\boldsymbol{\vartheta}) = \boldsymbol{C}\frac{\boldsymbol{S}_6 s^6 + \cdots + \boldsymbol{S}_1 s + \boldsymbol{S}_0}{\prod_{j=1}^{3}\left(s^2 + 2\zeta_j \omega_{n_j} s + \omega_{n_j}^2\right)}\boldsymbol{B}$$

6-th order transfer function

- 2^{nd} order modes (frequency damping)
- 3 modes (pitch, bending, and torsion)
- Relative degree zero

Frequency domain (s=iω)

Parameters ($\boldsymbol{\vartheta}$)

- Natural frequency
- Damping
- Numerator coefficients
  - matrices with 3 rows
  - one for each mode

Fitting the Model

Low order system

$$\boldsymbol{H}_{loes}(s|\boldsymbol{\vartheta}) = \boldsymbol{CHB}$$

Estimate states and output matrix ($\boldsymbol{C}$)

- Neglecting the inherent dynamics
  - Only done once
- Averaging over spatially distributed sensors
  - Asymptotically unbiased as the number of outputs is increased
  - Hence the Quasi maximum likelihood
- Reducing large number of outputs, to a small number with little sensor noise

Estimate transfer function parameters

- Similar to output error frequency domain system identification
- Results have a different interpretation
- Details in the paper

Estimating States and Output Matrix

Outputs by principal factor analysis

$$\boldsymbol{Z} \triangleq \boldsymbol{V}\boldsymbol{S}\widehat{\boldsymbol{X}}$$

- Generalized singular value decomposition of the data
  - Only states ($\widehat{\boldsymbol{X}}$) are complex numbers
  - Keeping 3 largest singular values
  - State correlation matrix, $\widehat{\boldsymbol{P}} = \boldsymbol{V}^{-1}\boldsymbol{Z}(\boldsymbol{V}^{-1}\boldsymbol{Z})^T$
- Sensor noise

$$\widehat{\boldsymbol{\Sigma}} = \mathrm{diag}\left(\boldsymbol{Z}\boldsymbol{Z}^T - \boldsymbol{V}\boldsymbol{S}\widehat{\boldsymbol{P}}\boldsymbol{S}\boldsymbol{V}^T\right)$$

  - Difference of the signal variance, and the variance from the decomposition
- State estimate
  - Expected value of the state given the measured output

$$\widehat{\boldsymbol{X}} = \left(\boldsymbol{I} + \widehat{\boldsymbol{P}}\boldsymbol{S}\boldsymbol{V}^T\widehat{\boldsymbol{\Sigma}}^{-1}\boldsymbol{V}\boldsymbol{S}\right)^{-1}\boldsymbol{S}\boldsymbol{V}^T\widehat{\boldsymbol{\Sigma}}\boldsymbol{Z}$$

    - Equivalent to Kalman filter as dynamics become infinitely fast
  - Least square estimates are shrunk to correct for sensor noise

$$\widehat{\boldsymbol{X}} = \left[\left(\boldsymbol{S}\boldsymbol{V}^T\widehat{\boldsymbol{\Sigma}}^{-1}\boldsymbol{V}\boldsymbol{S}\right)^{-1} + \widehat{\boldsymbol{P}}\right]^{-1} \cdot (\text{least squares})$$

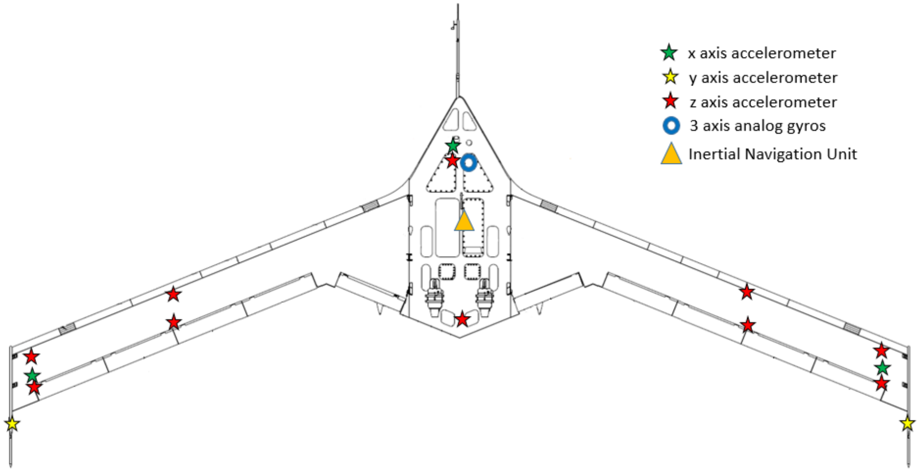
Inputs by least squares

- This is the minimum bias approach

Flight Test Results

Case	Airspeed, kn	Fuel, lb	Flight
1	80	62	16
2	110	23	34

- X-56A flex wing
- Pitch multisines
- Maneuver designed for the modes
- Above and below flutter speed



Selecting the number of modes

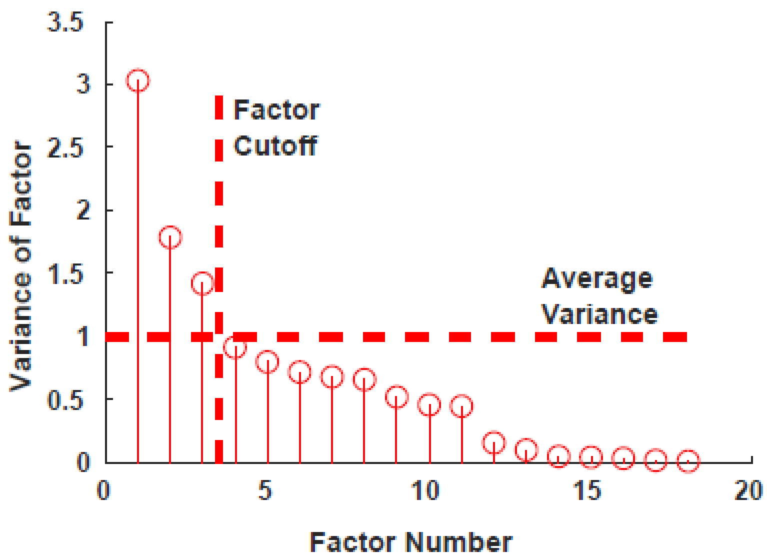
Examining diagonal of singular value matrix

3 modes is a reasonable solution

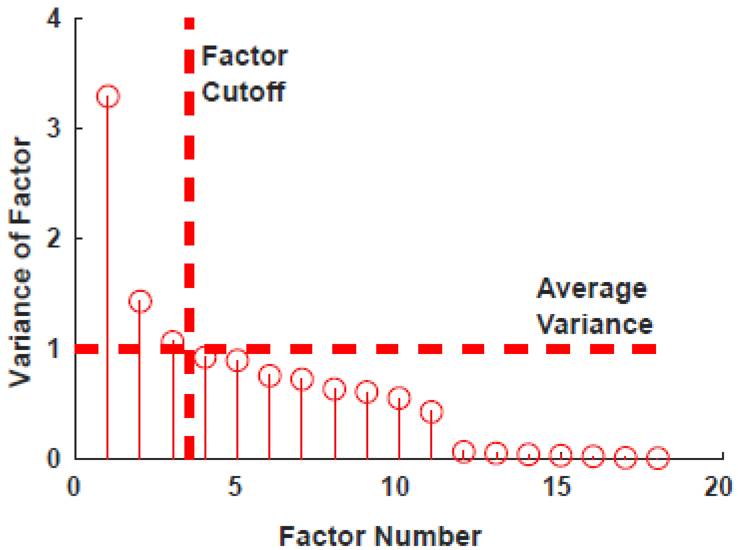
There could be as many as 11 modes

- This required more tuning of the model to flight condition

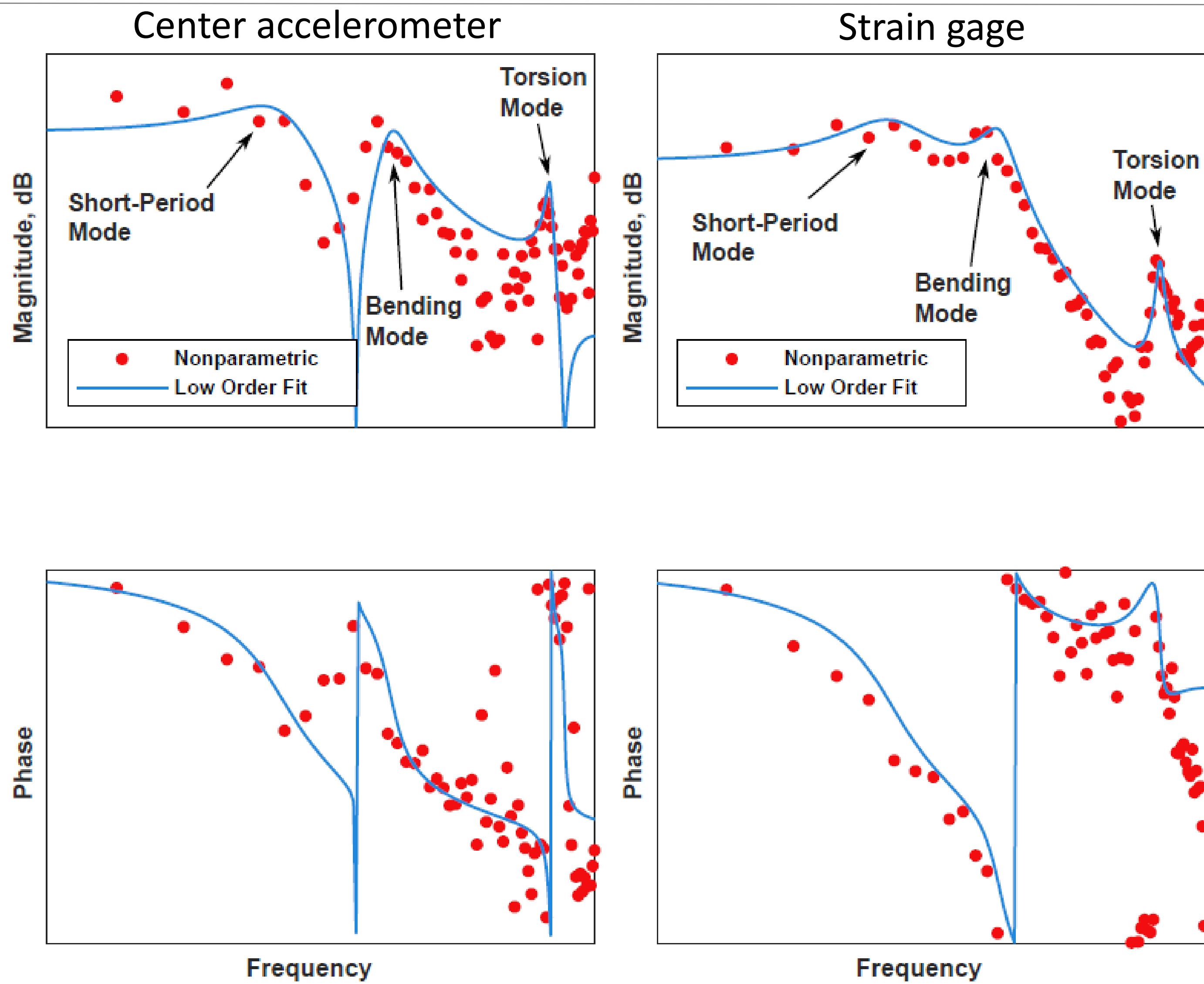
Low speed



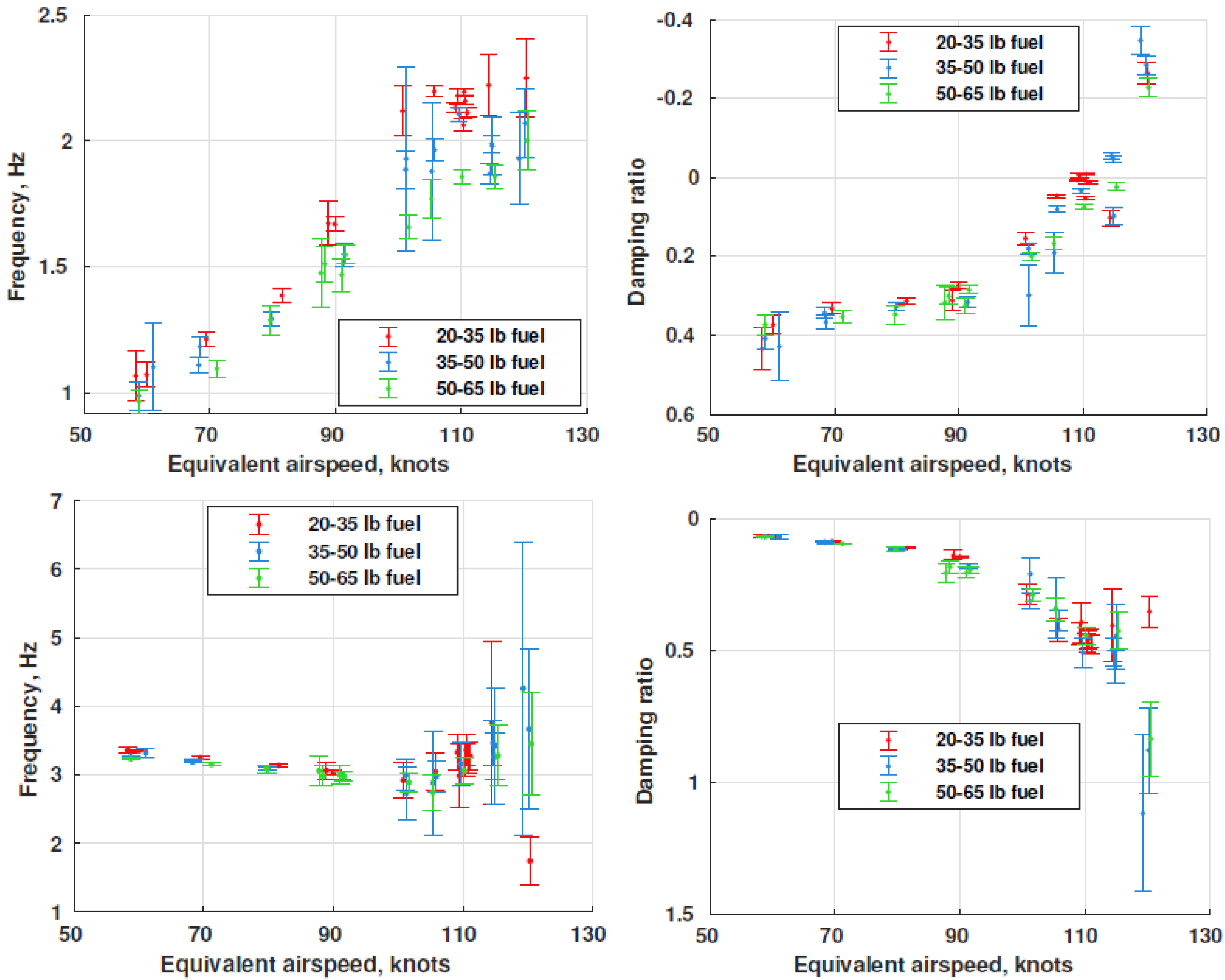
High speed



Low speed frequency response



Frequency and damping



Conclusions

Quasi maximum likelihood estimate of low order system

- Using large number of sensors to "average" out noise
- Consistent with full maximum likelihood as number of measurements is increased

Frequency-domain system identification

- Worked very well for open loop unstable aircraft

Method was reliable

- A single model structure was fit to most flight conditions without readjustments